5.1 Vector Addition/Subtraction (Graphically)

Vector **A** has a magnitude of 5 units and points to the east. Vector **B** has a magnitude of 3 units and points to the north. Using a ruler and protractor, find the magnitude and direction of the resultant vector **C.**

Use the head-to-tail method!
Magnitude → around 5.83 (units), rounded to 1 sig fig → **6** (units)

Check with Pythagorean Theorem: $5^2 + 3^2 = 34$, $\sqrt{34} = 5.83$

Direction: Inverse tangent: $tan^{-1}(\frac{3}{5}) = 30.96°$ above horizontal, or north of east.

Final answer (1 sig fig): **6 units, 30° north of east.**

Mr. Szwast walks 4 blocks, 30° north of east, then turns around and walks 6 blocks, 60° south of east. Using a ruler and protractor, find Mr. Szwast's total displacement.

Use the head-to-tail method!
Magnitude → around 7.21 blocks, rounded to 1 sig fig → **7 blocks**
The answer can be checked using the analytical method and resolving both vectors into x and y components.

Direction: around 26.3° south of east, rounded to one sig fig → **30°**
Final answer (1 sig fig): **7 blocks, 30° south of east.**

Vector **A** has a magnitude of 9 units, north. Vector **B** has a magnitude of 4 units, east. Find the magnitude of **A - B.** Use a ruler and protractor.

Use the head-to-tail method! Remember when subtracting a vector to reverse its direction.

Magnitude → around 9.8 units, rounded to 1 sig fig → **10 units**
The answer can be checked by resolving both vectors into x and y components, subtracting those of vector **B** from vector **A,** and using the Pythagorean theorem.

Final answer: **10 units**

A chicken (yes, a chicken) walks 2 miles, 45° north of east. The chicken then arrives at the farm and has lunch. After lunch, the chicken walks another 5 miles (very tenacious chicken), 60° north of west. Find the chicken's displacement, using a ruler and protractor.

Use the head-to-tail method!
The answer can be checked using the analytical method, as well.
Magnitude: around 5.8 miles → rounded to 1 sig fig → **6 miles**
Direction: 79.2° north of west → rounded to 1 sig fig → **80° north of west**
Final answer: **6 miles, 80° north of west**

5.2 Vector Addition Subtraction (Analytically)

A ladybug flies 27.8 m, 37.2° north of east. It then pauses on the windowsill, before flying another 24.6 m, 60.1° south of east. What is the direction and magnitude of the resultant vector?

Let the first vector (27.8 m, 37.2° north of east) be **A** and the second (24.6 m, 60.1° south of east) be **B.**
First, separate both vectors into x and y components. Let north and east be the positive directions while west and south will be written negative.

$A_x = 27.8\,cos(37.2)$ east

$A_y = 27.8\,sin(37.2)$ north

$B_x = 24.6cos(60.1)$ east

$B_y = 24.6sin(60.1)$ south (-)

Adding the two x-components and subtracting B_y from A_y gives:

$R_x = 34.406\,m$, $R_y = -4.518\,m$

Where **R** is the resultant vector. Using the Pythagorean theorem to combine the two gives:

$R = \sqrt{((34.41)^2 + (-4.518)^2)}$ = 34.7 m

Direction = $tan^{-1}(\frac{-4.60}{34.41}) = 7.61°$ south of east

Final answer: 34.7 m, 7.48° south of east

A wooden box experiences a force at two different angles:
F_1 = 10.0 N, 45.0° north of east
F_2 = 6.0 N, 30.0° south of east
Find the magnitude and direction of the resultant force.

Separate each force into x and y components. Let north and east be positive, while west and east will be written negative.

$F_{1x} = 10cos(45)$ east

$F_{1y} = 10sin(45)$ north

$F_{2x} = 6cos(30)$ east

$F_{2x} = 6sin(30)$ south (-)

Adding the x components and subtracting the y components gives:

$F_x = F_{1x} + F_{2x} = 12.27\,N$ north

$F_y = F_{1y} - F_{2y} = 4.07\,N$ east

Use the Pythagorean theorem to find the magnitude of the resultant vector:

$F = \sqrt{((12.27)^2 + (4.07)^2)}$ = 12.92 N → rounded to 2 sig figs → 13 N

Direction = $= tan^{-1}(\frac{4.60}{12.27}) = 18°$ north of east

Final answer: 13 N, 18° north of east

A woman is sailing in a boat. She first sails 43.2 m, 63.5° north of east. The instructions then said to travel 62.9 m, 31.0 north of east, but instead, the woman travels 62.9 m, 31.0 south of east! Where does she end up relative to where she started?

Let the first stretch of her trip be denoted **A (**43.2 m, 63.5° north of east) while the second stretch (62.9 m, 31.0 south of east) will be denoted **B.**

Let north and east be the positive directions, while west and south will be written negative. Resolving both vectors into components:

$A_x = 43.2cos(63.5)$ east

$A_y = 43.2sin(63.5)$ north

$B_x = 62.9cos(31.0)$ east

$B_y = 62.9sin(31.0)$ south (-)

Adding x-components and subtracting y-components:

$R_x = A_x + B_x = 73.201\, m$ east

$R_y = A_y - B_y = 6.23\, m$ north

Using the Pythagorean theorem to find the resultant:

$R = \sqrt{((73.201)^2 + (6.204)^2)}$ = 73.45 m → 73.5 m

Direction: $tan^{-1}(\frac{6.204}{73.201}) = 4.85°$ north of east

Final answer: 73.5 m, 4.85° north of east

Three forces act on a plastic container.

F_1 = 15.1 N, 0° (directly east)

F_2 = 10.2 N, 90° (directly north)

F_3 = 12.7 N, 210° (30° south of west)

Find the magnitude and direction of the resultant force.

Let north and east be the positive directions, while west and south will be written negative. Resolving all vectors into components:

$F_{1x} = 15.1cos(0) = 15.1\, N$ east,

$F_{1y} = 0\, N$

$F_{2x} = 0\, N$

$F_{2y} = 10.2\, N$ north

$F_{3x} = - 12.7cos(30) = - 11.0005\, N$ west

$F_{3y} = - 12.7sin(30) = - 6.35\, N\, south$

Adding all x and y components:

$F_x = F_{1x} + F_{2x} + F_{3x}$ = 4.099 N east

$F_y = F_{1y} + F_{2y} + F_{3y} = 3.85\ N$ north

Using Pythagorean theorem to find the resultant:

$F = \sqrt{((4.099)^2 + (3.85)^2)}$ = 5.6258 N → rounded to 3 sig figs → 5.63 N

Direction = $tan^{-1}(\frac{3.85}{4.099}) = 43.7°$ north of east

Final answer: 5.63 N, 43.7° north of east

5.3 Projectile Motion

A man shoots an arrow with velocity of 25 m/s at an angle of 20 degrees 8.0 meters above the ground. What is the speed just before it hits the ground?

$\theta = 35.0$

$x = 20.0\ m$

$v_{0y} = 25\ sin\ 20$

$v_{0x} = v_x = 25\ cos\ 20$

$$v^2 = v_0^2 + 2a(y - y_0)$$

$$v^2 = \sqrt{(25\ sin\ 20)^2 + 2(-9.80)(8.0)}$$

$$v_y = 15.17\ m/s$$

$$v = \sqrt{v_y^2 + v_x^2}$$

$$v = \sqrt{(15.17)^2 + (25\ cos\ 20)^2} \approx 28\ m/s$$

Ryan Mansour is playing Overwatch when he rages and throws his computer into a 1.00 meter tall trash can downstairs. He throws it horizontally, with a velocity of 2.50 m/s, at a height of 20.6 meters. How far must the trash be placed?

$v_{0y} = 0\ m/s$

$v_{0x} = v_x = 2.50\ m/s$

$y_0 = 20.6\ m$

$y = 1.00\ m$

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2$$

$$1.00 = 20.6 + 0 - 4.9t^2$$

$$-19.6 = -4.9t^2$$

$$t = \sqrt{\frac{-19.6}{-4.9}} = 2.00$$

$$x = v_x t$$

$$x = (2.50)(2.00) = 5.00\ m$$

Veer kicks a soccer ball with a velocity v and an angle of 35.0 degrees. The ball lands 20.0 meters away. What is the velocity Veer kicked it at?

$\theta = 35.0$

$x = 20.0\ m$

$v_{0y} = v\ sin\ 35$

$v_{0x} = v_x = v\ cos\ 35$

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2$$

$$0 = 0 + vtsin\ 35 - 4.9t^2$$

$$t(v\ sin\ 35 - 4.9t) = 0$$

$$t = 0 \text{ or } t = \frac{v\ sin\ 35}{4.9}$$

$$x = v_x t$$

$$20 = (v\ cos\ 35)\ (\frac{v\ sin\ 35}{4.9})$$

$$v^2 = \frac{(4.9)(20)}{(sin\ 35)(cos\ 35)}$$

$$v = \sqrt{\frac{(4.9)(20)}{(sin\ 35)(cos\ 35)}}$$

$$v \approx 14.4\ m/s$$

Veer is facing North and also attempts to kick the soccer ball back, at an angle of 40.0 degrees at a velocity of 18 m/s North. While kicking it, he accidentally also makes it go 2.50 m/s west. What is the distance of the ball relative to Veer?

$\theta = 40.0$

$v_{0y} = 18 \sin 40$

$v_{0x} = v_x = 18 \cos 40$

$v_z = 2.50\, m/s$

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2$$

$$0 = 0 + 18t \sin 35 - 4.9t^2$$

$$t(18 \sin 40 - 4.9t) = 0$$

$$t = 0 \text{ or } t = \frac{18 \sin 40}{4.9} \approx 2.6312$$

$$x = v_x t$$

$$x = (18 \cos 35)(2.6312) = 38.796$$

$$z = v_z t$$

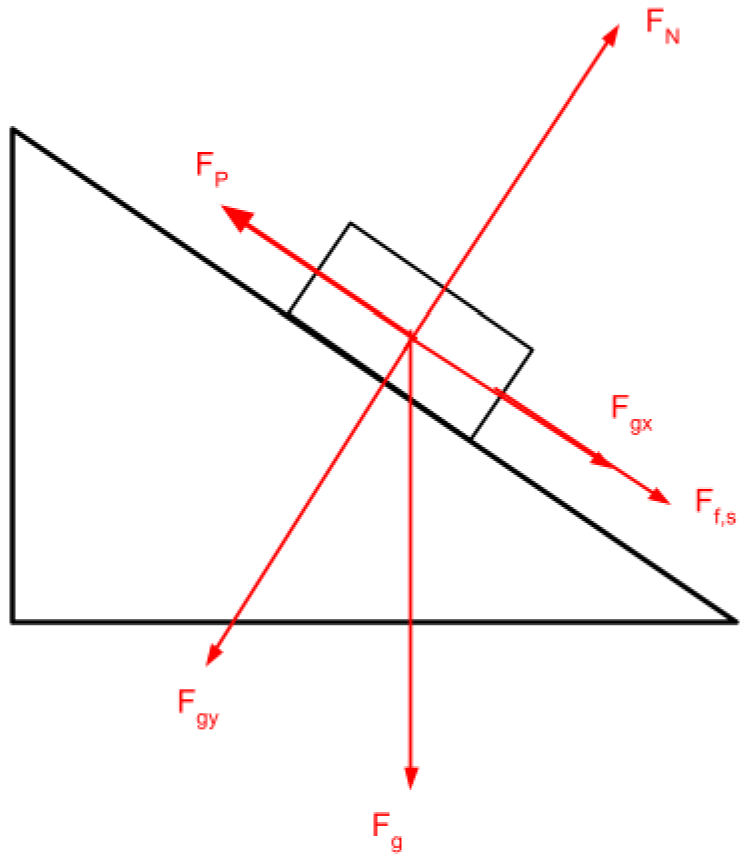
$$z = (2.50)(2.6312) = 6.578$$

$$d = \sqrt{x^2 + z^2}$$

$$d = \sqrt{38.796^2 + 6.578^2} = 36.4\, m$$

5.4 Inclined Planes

Draw a free body diagram of a stationary block on an inclined plane with an applied force upwards.



A 10.0 kg block is stationary on an inclined plane with an angle of 22.4 degrees. What is the force of friction on the block?

$F_g = 9.80 \times 10.0 = 98.0\ N$

$F_{gx} = 98.0 \sin 22.4 N$

$F_{Fs} = F_{gx} = 98.0 \sin 22.4 \approx 37.3\ N$

A 30.0 kg block is moving downwards on an inclined plane with an angle of 35.0 degrees. If the coefficient of friction is 0.250, what is the magnitude of the acceleration of the box?

$F_g = 9.80 \times 30.0 = 294\ N$

$F_{gx} = 294 \sin 35.0 = 168.63\ N$

$F_{gy} = 294 \cos 35.0\ N = F_N = 240.83\ N$

$F_{Fk} = \mu_k F_N = (.250)(240.83) = 60.2075 N$

$a = \frac{F}{m} = \frac{168.63 - 60.2075}{30.0} = 3.61\ m/s^2$

$F_g = 9.80 \times 32.0 = 313.6 N$

$F_{gx} = 313.6 \sin 40.0 = 201.58\ N$

$F_{gy} = 313.6 \cos 40.0\ N \ = \ F_N = 240.23\ N$

$F_N = 400.\ N - 201.58\ N \ = 198.42\ N$

$\mu_s = \frac{198.42N}{240.23N} = 0.826$

5.5 Simple Harmonic Motion

A block is hung from a vertical spring with a spring constant 250. N/m. When it is hung on the spring, the spring stretches 0.042 m. If the spring is forced to oscillate with the block attached to it, what is the period of oscillation?

$k = 250.\ N/m$

$\Delta x = 0.042\ m$

$F_s = ?$

$a = -9.81\ m/s$

$g = 9.81\ m/s$

$m = ?$

$T = ?$

$F_s = -k\Delta x \leftarrow$ Hooke's Law

$= -(250.)(0.042)$

$= -10.5$

$F_s = F_g \leftarrow$ gravity pulls the block down

$-10.5 = ma$

$-10.5 = m(-9.81)$

$1.070336391 = m \leftarrow$ solving for m, mass

$T = (2\pi)\sqrt{\frac{m}{k}}$

$T = (2\pi)\sqrt{\frac{1.070336391}{250.}}$

$T = 0.41112133\ s$

$T = 0.41\ s$

Given that the frequency of oscillation is 3.87 Hz and the gravitational field strength in this region is 9.78 N/kg, what is the length of the pendulum (in centimeters)?

$f = 3.87\ Hz$

$g = 9.78\ kg$

$T = ?$

$L = ?$

$T = 1/f$

$= 1/3.87$

$= 0.264550265$

$T = (2\pi)\sqrt{\frac{L}{g}}$

$(\frac{T}{(2\pi)})^2(g) = L \leftarrow$ rearranging

$(\frac{0.26455...}{(2\pi)})^2(9.78) = 0.017337...$

converting to cm and rounding $\rightarrow$ $L = 1.73\ cm$

If a block of mass 8.95 kg is attached to a horizontal spring (k = 175 N/m) that is stretched 87.0 cm, calculate the magnitude of the restoring force.

$m = 8.95\ kg$

$k = 175\ N/m$

$\Delta x = 87.0\ cm = 0.870\ m$

$F_s = ?$

$F_s = -k\Delta x$

$|F_s| = k\Delta x$

$= (175)(0.870)$

$= 152.25$

Rounding to 3 significant figures: $152\ N$

A mass m is oscillating freely on a spring. When the mass is 0.76 kg, the period is 0.85 seconds. An unknown mass on the spring has a period of 1.25 seconds.
a) Find the spring constant of the spring, k.
b) Find the unknown mass.

$m_1 = 0.76\ kg$

$T_1 = 0.85\ s$

$m_2 = ?$

$T_2 = 1.25\ s$

$k = ?$

$T_1 = (2\pi)\sqrt{\frac{m_1}{k}}$

$(\frac{T_1}{(2\pi)})^2 = \frac{m_1}{k}$

$k = \frac{m_1}{(\frac{T_1}{(2\pi)})^2}$

$k = \frac{0.76}{(\frac{0.85}{(2\pi)})^2} = 41.52747042$

a) $k = 42\ N/m$

$T_2 = (2\pi)\sqrt{\frac{m_2}{k}}$

$(\frac{T_2}{(2\pi)})^2(k) = m_2$

$(\frac{1.25}{(2\pi)})^2(41.527...) = m_2$

$1.643598616 = m_2$

b) $m_2 = 1.6\ kg$